Mock AIME One

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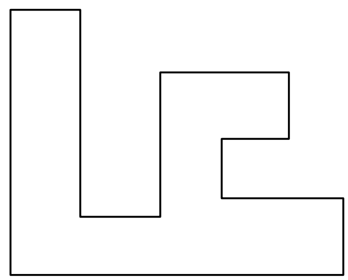
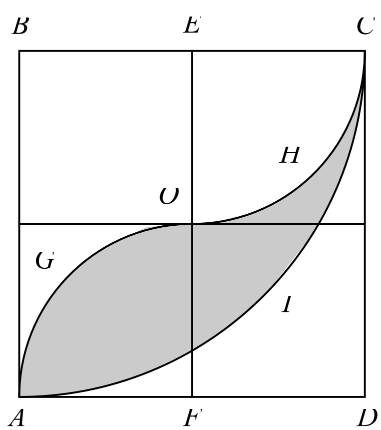
Problem 1

To obtain the perimeter of the shape where all angles are right, at least how many segments must be measured?



Problem 2

ABCD is a square of side length 20 cm. *O* is its center. *E* and *F* are the midpoints of *BC* and *AD* respectively. Arc *OHC* is drawn with *E* as its center, and arc *OGA* is drawn with *F* as its center. Arc *AIC* is drawn with *B* as its center. Let *s* the area of the shaded region in cm^2. Find $\lfloor s \rfloor$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to *x*.



Problem 3

Three boys *A, B,* and *C* sit around a round table in that order. *A* has a ball in his hand. Starting from *A* the boy having the ball passes the ball to either of the other two boys. After 5 passes the ball goes back to *A*. How many different ways can the ball be passed? For example, the following are two different ways:

$$A \to B \to A \to C \to B \to A \text{ and } A \to C \to B \to C \to B \to A.$$

Problem 4

Divide 2008 numbers from 1 to 2008 into 8 groups with each group containing 251 numbers. Let x_i be the median of the ith group, where $i = 1, 2, \cdots, 8$. Let

$$S = \sum_{i=1}^{8} x_i .$$

Let m be the minimum value and M be the maximum value of S. Find $\frac{M - m}{10}$.

Problem 5

Let $x = \left(\sqrt{5} + 2\right)^{2009}$, find $x\left(x - \lfloor x \rfloor\right)$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

Problem 7

It is given that x and y are real numbers such that $x^2 - 6x + 9y^2 = 16$. The maximum value of $x + y$ can be expressed as $a + \frac{\sqrt{b}}{c}$ where a, b, and c are positive integers with b and c relatively prime. Find b.

Problem 9

I have a deck of 999 cards numbered in order from 1 to 999. I throw away the top card (card 1), and place the next card (card 2) at the bottom of the deck. Then I throw away the top card of the remaining deck (card 3), and place the next card (card 4) at the bottom of the deck. We repeat the procedure: throwing away the top card, and placing the next card at the bottom of the deck. Eventually, we have one card left. Find the number of the card.

Problem 11

Let n be a natural number. A permutation $\left(a_1, a_2, a_3, \cdots, a_n\right)$ of $\left(1, 2, 3, \cdots, n\right)$ is of *good order* if there is one and ***only*** one number j with $1 \le j \le n - 1$ such that $a_{j+1} < a_j$. For example, for $n = 4$, $\left(1, 3, 2, 4\right)$ and $\left(2, 3, 1, 4\right)$ are of good order, while $\left(1, 4, 3, 2\right)$, $\left(4, 3, 2, 1\right)$ and $\left(1, 2, 3, 4\right)$ are not. Let p_n be the number of permutations of $\left(1, 2, 3, \cdots, n\right)$ which are of good order. Find p_9.

Problem 13

The minimum value of $f\left(x\right) = \sqrt{x^4 - x^2 + 1} + \sqrt{x^4 - 5x^2 - 4x + 13}$ where x is a real number can be expressed as $\sqrt{m}$ where m is an integer. Find m.

