The theme for this session is solid geometry, the geometry of three dimensional objects. Most AIMEs have one or more solid geometry problems. The 2009 AIMEs were unusual in not having any. Arguably this guarantees that such problems will appear in 2010. Plus they appear as some of the harder AMC problems.

The challenge with solid geometry problems is typically understanding and visualizing the problem. With planar geometry (two dimensions) one can draw a diagram. But that is not so easy with three dimensions. Mental visualization can help, but the pressure of the contest setting often interferes with achieving a suitably contemplative mood. But if one can visualize the problem, the actual solution is often quite simple.

Following are twelve warm up problems, some of which you may have seen before. The first few are easier, later ones are harder. Please look them over and determine how you would approach their solution. Try to find quick, intuitive solutions suitable for the limited time available on AMC and AIME. We will discuss solutions of these at the beginning of the session.

Answers to the warm up problems are listed below. Note the purpose of our discussion will be how one finds the answer, not the answer itself.

1. $2\sqrt{2}$
2. 8
3. 4000
4. 729
5. $\frac{\sqrt{6}}{2}$
6. 014
7. 784
8. $4+2\sqrt{2}$
9. 14
10. 080
11. 300
12. 2

**1.** What is the volume of a cube whose surface area is twice that of a cube with volume 1?

(A) $\sqrt{2}$ (B) $2$ (C) $2\sqrt{2}$ (D) $4$ (E) $8$

**2.** A sphere is inscribed in a cube that has a surface area of 24 square meters. A second cube is then inscribed within the sphere. What is the surface area in square meters of the inner cube?

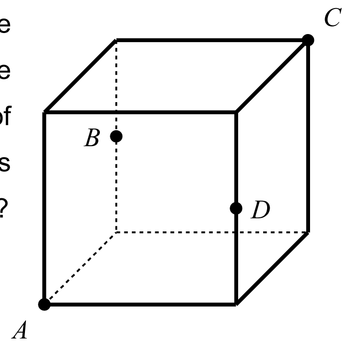
(A) 3 (B) 6 (C) 8 (D) 9 (E) 12

**3.** A cone-shaped mountain has its base on the ocean floor and has a height of 8000 feet. The top $\frac{1}{8}$ of the volume of the mountain is above water. What is the depth of the ocean at the base of the mountain, in feet?

(A) $4000$ (B) $2000\,(4-\sqrt{2})$ (C) $6000$ (D) $6400$ (D) $7000$

**4.** A block of cheese in the shape of a rectangular solid measures 10 cm by 13 cm by 14 cm. Ten slices are cut from the cheese. Each slice has a width of 1 cm and is cut parallel to one face of the cheese. The individual slices are not necessarily parallel to each other. What is the maximum possible volume in cubic cm of the remaining block of cheese after ten slices have been cut off?

**5.** A cube with side length 1 is sliced by a plane that passes through two diagonally opposite vertices $A$ and $C$ and the midpoints $B$ and $D$ of two opposite edges not containing $A$ or $C$, as shown. What is the area of quadrilateral $ABCD$?




(A) $\frac{\sqrt{6}}{2}$ (B) $\frac{5}{4}$ (C) $\sqrt{2}$ (D) $\frac{3}{2}$ (E) $\sqrt{3}$

**6.** A right circular cone has base radius $r$ and height $h$. The cone lies on its side on a flat table. As the cone rolls on the surface of the table without slipping, the point where the cone's base meets the table traces a circular arc centered at the point where the vertex touches the table. The cone first returns to its original position on the table after making 17 complete rotations. The value of $h/r$ can be written in the form $m\sqrt{n}$, where $m$ and $n$ are positive integers and $n$ is not divisible by the square of any prime. Find $m+n$.

**7.** A pyramid has a square base $ABCD$ and vertex $E$. The area of square $ABCD$ is 196, and the areas of $\triangle ABE$ and $\triangle CDE$ are 105 and 91, respectively. What is the volume of the pyramid?

(A) $392$ (B) $196\sqrt{6}$ (C) $392\sqrt{2}$ (D) $392\sqrt{3}$ (E) $784$

**8.** A pyramid with a square base is cut by a plane that is parallel to its base and is 2 units from the base. The surface area of the smaller pyramid that is cut from the top is half the surface area of the original pyramid. What is the altitude of the original pyramid?

(A) 2 (B) $2+\sqrt{2}$ (C) $1+2\sqrt{2}$ (D) 4 (E) $4+2\sqrt{2}$

**9.** A regular octahedron has side length 1. A plane parallel to two of its opposite faces cuts the octahedron into two congruent solids. The polygon formed by the intersection of the plane and the octahedron has area $\frac{a\sqrt{b}}{c}$, where $a$, $b$ and c are positive integers, $a$ and $c$ are relatively prime, and $b$ is not divisible by the square of any prime. What is $a+b+c$ ?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

**10.** A square pyramid with base $ABCD$ and vertex $E$ has eight edges of length 4. A plane passes through the midpoints of $\overline{AE}$, $\overline{BC}$, and $\overline{CD}$. The plane's intersection with the pyramid has an area that can be expressed as $\sqrt{p}$. Find $p$.

**11.** A convex polyhedron $Q$ has vertices $V_1, V_2, \ldots, V_n$, and 100 edges. The polyhedron is cut by planes $P_1, P_2, \ldots, P_n$ in such a way that plane $P_k$ cuts only those edges that meet at vertex $V_k$. In addition, no two planes intersect inside or on $Q$. The cuts produce $n$ pyramids and a new polyhedron $R$. How many edges does $R$ have?

(A) 200 (B) 2n (C) 300 (D) 400 (E) 4n

**12.** Points $A$, $B$, $C$, $D$, and $E$ are located in 3-dimensional space with $\overline{AB}=\overline{BC}=\overline{CD}=\overline{DE}=\overline{EA}=2$ and $\measuredangle ABC=\measuredangle CDE=\measuredangle DEA=90^\circ$. The plane of $\triangle ABC$ is parallel to $\overline{DE}$. What is the area of $\triangle BDE$?

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{5}$ (E) $\sqrt{6}$