

Answers to the problems are listed below. Note the purpose of our discussion will be how one finds the answer, not the answer itself.

1. 18
2. 36
3. 3 : 1
4. 505
5. $3 + \frac{\sqrt{69}}{3}$
6. 813

1. A solid box is 15 cm by 10 cm by 8 cm. A new solid is formed by removing a cube 3 cm on a side from each corner of this box. What percent of the original volume is removed?

- (A) 4.5 (B) 9 (C) 12 (D) 18 (E) 24

2. A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

- (A) 10 (B) 25 (C) 36 (D) 50 (E) 60

3. An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cone's height to its radius? (Note: A cone with radius r and height h has volume $\pi r^2 h/3$, and a sphere with radius r has volume $4\pi r^3/3$.)

- (A) 2 : 1 (B) 3 : 1 (C) 4 : 1 (D) 16 : 3 (E) 6 : 1

4. Consider the set of points that are inside or within one unit of a rectangular parallelepiped (box) that measures 3 by 4 by 5 units. Given that the volume of this set is $\frac{(m+n)\pi}{p}$, where m , n , and p are positive integers, and n and p are relatively prime, find $m+n+p$.

5. Three mutually tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

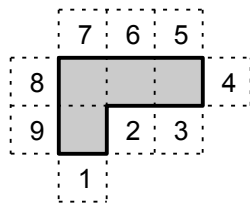
- (A) $3 + \frac{\sqrt{30}}{2}$ (B) $3 + \frac{\sqrt{69}}{3}$ (C) $3 + \frac{\sqrt{123}}{4}$ (D) $\frac{52}{9}$ (E) $3 + 2\sqrt{2}$

6. A unicorn is tethered by a 20-foot silver rope to the base of a magician's cylindrical tower whose radius is 8 feet. The rope is attached to the tower at ground level and to the unicorn at a height of 4 feet. The unicorn has pulled the rope taut, the end of the rope is 4 feet from the nearest point on the tower, and the length of the rope that is touching the tower is $\frac{a-\sqrt{b}}{c}$ feet, where a , b , and c are positive integers, and c is prime. Find $a+b+c$.

Answers to the problems are listed below. Note the purpose of our discussion will be how one finds the answer, not the answer itself.

1. 6
2. 028
3. 240
4. 6
5. 216
6. 2 : 1

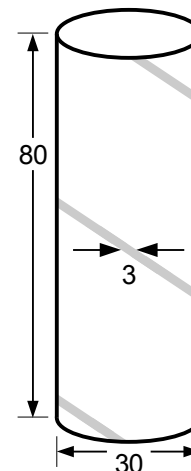
1. The polygon enclosed by the solid lines in the figure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?



- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. In a regular tetrahedron the centers of the four faces are the vertices of a smaller tetrahedron. The ratio of the volume of the smaller tetrahedron to that of the larger is m/n , where m and n are relatively prime positive integers. Find $m + n$.

3. A white cylindrical silo has a diameter of 30 feet and a height of 80 feet. A red stripe with a horizontal width of 3 feet is painted on the silo, as shown, making two complete revolutions around it. What is the area of the stripe in square feet?



- (A) 120 (B) 180 (C) 240 (D) 360 (E) 480

4. A truncated cone has horizontal bases with radii 18 and 2. A sphere is tangent to the top, bottom, and lateral surface of the truncated cone. What is the radius of the sphere?

- (A) 6 (B) $4\sqrt{5}$ (C) 9 (D) 10 (E) $6\sqrt{3}$

5. A cylindrical log has diameter 12 inches. A wedge is cut from the log by making two planar cuts that go entirely through the log. The first is perpendicular to the axis of the cylinder, and the plane of the second cut forms a 45° angle with the plane of the first cut. The intersection of these two planes has exactly one point in common with the log. The number of cubic inches in the wedge can be expressed as $n\pi$, where n is a positive integer. Find n .

6. Four planes divide a cube into five tetrahedrons. Four of the tetrahedrons are congruent, the fifth is regular. What is the ratio of the largest tetrahedron's volume to that of the smallest?