colorado math circle

Math Battle

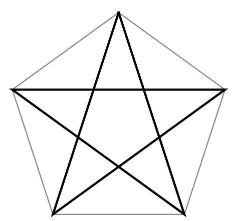
Galois Group

December 17, 2011

1. Given any 10 line segments, are there always 3 segments that can form a triangle?

2. Prove that the fraction $\frac{3n+8}{5n+13}$ cannot be reduced for any positive integer n.

3. Three dimes are placed on the vertices of a pentagon. You are allowed to move a dime along any diagonal of the pentagon to any unoccupied vertex. Is it possible that after several such moves one of the dimes occupies its original position while the other two have switched places?



4. Are there two consecutive numbers such that the sums of their digits are both divisible by 7?

5. Can you draw a continuous curved line that crosses each line segment in Figure 1 exactly once? An example is shown in Figure 2.

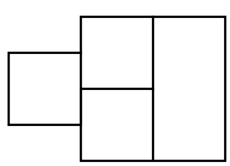


Figure 1

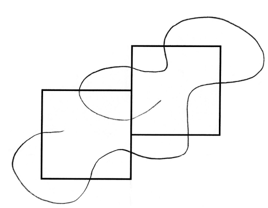


Figure 2

6. In a warehouse N boxes marked 1 through N are stacked in two piles. A forklift can take several boxes from the top of one pile and place them on top of the other pile. Prove that all the boxes can be stacked in one pile in decreasing order of their numbers (with box 1 on top) using at most $2N-1$ operations of the forklift.

Problems adapted from Mathematical Circles (Russian Experience) by D. Fomin, S. Genkin, I. Itenberg, American Mathematical Society, 1996.