Counting and Probability

Colorado Math Circle

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Chapter MATHCOUNTS

1. A jar contains 10 red, 7 blue and 5 yellow marbles. Blue marbles are then added in order to change the probability of randomly selecting a blue marble from the jar to "greater than 1/2". What is the least number of blue marbles that must be added?

   Chapter Sprint 2002 #20

2. What is the probability that the square root of a randomly selected two-digit whole number is less than eight? Express your answer as a common fraction.

   Chapter Sprint 2002 #23

3. Each day, two out of the three teams in a class are randomly selected to participate in a MATHCOUNTS trial competition. What is the probability that Team A is selected on at least two of the next three days? Express your answer as a common fraction.

   Chapter Sprint 2003 #29

4. Five balls are numbered 1 through 5 and placed in a bowl. Josh will randomly choose a ball from the bowl, look at its number and then put it back into the bowl. Then Josh will again randomly choose a ball from the bowl and look at its number. What is the probability that the product of the two numbers will be even and greater than 10? Express your answer as a common fraction.

   Chapter Sprint 2006 #22

5. Four couples are at a party. Four people of the eight are randomly selected to win a prize. No person can win more than one prize. What is the probability that both members of at least one couple win a prize? Express your answer as a common fraction.

   Chapter Sprint 2007 #28

6. A jar contains two red marbles, three green marbles, ten white marbles and no other marbles. Two marbles are randomly drawn from this jar without replacement. What is the probability that these two marbles drawn will both be red? Express your answer as a common fraction.

   Chapter Sprint 2008 #27

7. In a 7-by-7 checkerboard, two unit squares will be chosen at random and without replacement. What is the probability that the two squares are adjacent to each other (share a side)? Express your answer as a common fraction.

   Chapter Sprint 2009 #29

8. Six boys and six girls are seated randomly in a row of 12 chairs. What is the probability that no two boys are seated next to one another and no two girls are seated next to one another? Express your answer as a common fraction.

   Chapter Target 2007 #7

9. If Ella rolls a standard six-sided die until she rolls the same number on consecutive rolls, what is the probability that her 10th roll is her last roll? Express your answer as a decimal to the nearest thousandth.

   Chapter Team 2005 #9

State MATHCOUNTS

10. Container I holds 8 red balls and 4 green balls; containers II and III each hold 2 red balls and 4 green balls. A container is selected at random and then a ball is randomly selected from that container. What is the probability that the ball selected is green? Express your answer as a common fraction.

    State Sprint 2003 #21

11. Of the final five contestants in a television show, three are female and two are male. If two are chosen randomly to be the final contestants, what is the probability that both are female? Express your answer as a common fraction.

    State Sprint 2004 #14

12. There are only red marbles and green marbles in a bag. The ratio of red marbles to green marbles in the bag is 4:7. Julia then adds 90 red marbles and 36 green marbles to the bag, which makes the probability of selecting a red marble from the bag on a random draw equal to 1/2 . How many total marbles are in the bag after Julia has added the 126 marbles?

    State Sprint 2005 #24

13. Six students are being grouped into three pairs to work on a science lab. How many different combinations of three pairs are possible?

    State Sprint 2009 #25

14. Points X, Y and Z lie on the sides of triangle ABC so that segments AX, BY and CZ, if drawn, would intersect at one interior point P. Using 3 of these 7 points at a time as vertices, how many triangles can be formed?

    State Sprint 2009 #28

15. While staying in a 15-story hotel, Polya plays the following game. She enters an elevator on the 6th floor. She flips a fair coin five times to determine her next five stops. Each time she flips heads, she goes up one floor. Each time she flips tails, she goes down one floor.What is the probability that each of her next five stops is on the 7th floor or higher? Express your answer as a common fraction.

    State Target 2005 #7

16. In a math class, 12 out of 15 girls are freshmen and 11 out of 15 boys are freshmen. What is the probability that in a randomly selected group of five students from the class, there will be two freshmen girls and three freshmen boys? Express your answer as a decimal to the nearest thousandth.

    State Team 2006 #10

National MATHCOUNTS

17. There are five chairs on stage, arranged in a straight line parallel to the front of the stage. Big Kahuna, Major Domo, Em See, Lou Tenant, and Captain Marv El will occupy the five chairs, but Major Domo cannot sit next to Big Kahuna. How many seating arrangements are possible?

    National Sprint 1997 #18

18. The ten letters of the word MATHCOUNTS are written on cards, one letter per card, and placed in a basket. When two cards are picked at random without replacement, what is the probability that the first card chosen has a vowel on it and the second card chosen has a consonant? Express your answer as a common fraction.

    National Sprint 1998 #24

19. A set of cards consists of 8 red and 7 black cards. Three cards are dealt at random without replacement. What is the probability that the three cards dealt are the same color? Express your answer as a common fraction.

    National Sprint 1999 #6

20. The game of Rock, Paper, Scissors is played by three players. Each player begins by making a fist; then, on the count of four, shows either four fingers (Paper), two fingers (Scissors) or a fist (Rock). What is the probability that exactly two players show the same sign? Express your answer as a common fraction.

    National Target 1998 #3

21. How many even integers between 8000 and 9999 have digits that are all distinct?

    National Team 1995 #6

22. A mayonnaise jar contains 6 red marbles and 4 blue marbles. A jelly jar contains 2 red marbles and 5 blue marbles. One marble is randomly selected from the mayonnaise jar and placed in the jelly jar. A marble is then randomly selected from the jelly jar. What is the probability that the selected marble is red? Express your answer as a common fraction.

    National Team 1998 #6

AMC10

23. A point (x, y) is randomly picked from inside the rectangle with vertices (0, 0), (4, 0), (4, 1), and (0, 1). What is the probability that $x < y$?

    2003 AMC10a #12

24. A bag contains two red beads and two green beads. You reach into the bag and pull out a bead, replacing it with a red bead regardless of the color you pulled out. What is the probability that all beads in the bag are red after three such replacements?

    2003 AMC10b #21

25. Coin A is flipped three times and coin B is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?

    2004 AMC10a #10

26. Each face of a cube is painted either red or blue, each with probability 1/2. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

    2004 AMC10b #23

27. Three tiles are marked X and two other tiles are marked O. The five tiles are randomly arranged in a row. What is the probability that the arrangement reads XOXOX?

    2005 AMC10a #9

28. Team A and team B play a series. The first team to win three games wins the series. Each team is equally likely to win each game, there are no ties, and the outcomes of the individual games are independent. If team B wins the second game and team A wins the series, what is the probability that team B wins the first game?

    2005 AMC10a #18

29. Twelve fair dice are rolled. What is the probability that the product of the numbers on the top faces is prime? Express your answer using exponents.

    2005 AMC10b #12

30. Forty slips are placed into a hat, each bearing a number 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10, with each number entered on four slips. Four slips are drawn from the hat at random and without replacement. Let p be the probability that all four slips bear the same number. Let q be the probability that two of the slips bear a number a and the other two bear a number $b \neq a$. What is the value of q/p?

    2005 AMC10b #21

31. For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5 and 6 on each die are in the ratio 1:2:3:4:5:6. What is the probability of rolling a total of 7 on the two dice?

    2006 AMC10b #21