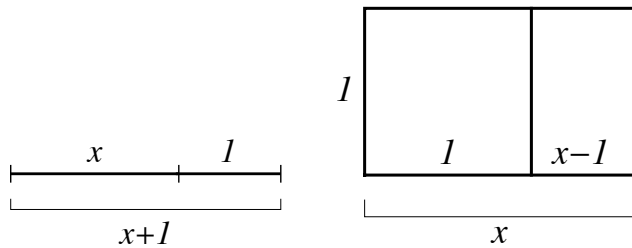


The Golden Ratio and Penrose Tilings

Colorado Math Circle
Galois Group

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*The good, of course, is always beautiful,
and the beautiful never lacks proportion.*

—Plato (400 BC)

*A straight line is said to have been cut in extreme and mean ratio
when, as the whole line is to the greater segment,
so is the greater to the less.*

—Euclid (300 BC)

*Whoever cultivates the golden mean avoids both
the poverty of a hovel and the envy of a palace.*

—Horace (65 BC)

1. Find the exact value of the golden ratio ϕ by solving the following proportions for the *golden section* and *golden rectangle*, respectively. (See figures at left.)

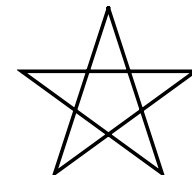
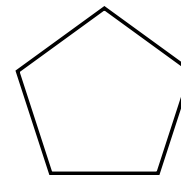
$$\frac{x}{1} = \frac{x+1}{x}$$

$$\frac{x}{1} = \frac{1}{x-1}$$

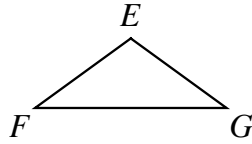
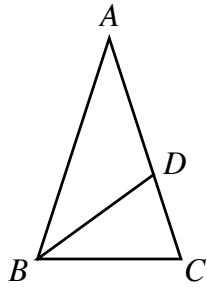
2. Confirm the following identities.

(a) $\phi^2 = \phi + 1$

(b) $\frac{1}{\phi} = \phi - 1$



3. Find the measure of an interior angle of a regular pentagon.
4. Find the measures of all the interior angles of a regular pentagram.

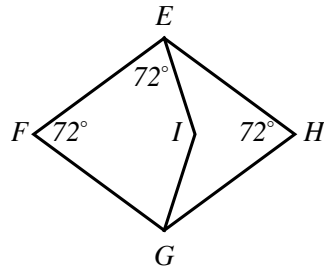
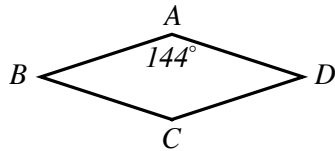


5. Consider the *golden triangle* shown above. Sides $AB = AC$ and angle $BAC = 36^\circ$. Let $BC = 1$.

- Find the measures of the base angles ABC and ACB .
- If BD bisects $\angle ABC$, find the measures of $\angle ADB$ and $\angle BDC$.
- What is the length of AB ?

6. Consider the *golden gnomon* shown above. Sides $EF = EG$ and angle $FEG = 108^\circ$. Let $EF = 1$.

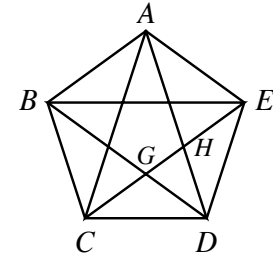
- Find the measures of angles EFG and EGF .
- What is the length of FG ?



7. Consider rhombuses $ABCD$ and $EFGH$ shown above.

- If $AB = 1$, what is the length of AC ?
- If $EF = 1$, what is the length of EI ? What is the length of FH ?

8. Consider the regular pentagon $ABCDE$ with side length 1, shown at right. Find the following ratios.



- CE/CH
- CH/CG
- CG/GH

9. Let

$$x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$$

Find the value of x .

10. Let

$$x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

Find the value of x .

11. A *regular tessellation* tiles the plane with congruent regular polygons. There are exactly three types, composed of triangles, squares or hexagons. A *semiregular tessellation* uses a variety of regular polygons, with the arrangement at every vertex identical. Can you find all eight semiregular tessellations?

12. Two different positive numbers a and b each differ from their reciprocals by 1. What is $a + b$? (AMC12 2002A)

13. A *Kepler Triangle* is a right triangle with edge lengths in geometric progression. Let the length of the shorter leg equal 1. Find the lengths of the other two sides.

14. The ratio of the length of a rectangle to the width is the same as the ratio of the diagonal to the length. If the width is 2, how many units are in the length of the diagonal?

15. Equilateral triangle ABC is inscribed in a circle. Points D and E are the midpoints of AB and AC , respectively. Extend \overline{DE} so that it intersects the circle at F . Find the ratio DE/EF .