

An Introduction to Recursion

Colorado Math Circle

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“To understand recursion, one must first understand recursion.”

A **recursive process** is one in which objects are defined in terms of other objects of the same type.

A **recurrence relation** is an equation that defines each term of a sequence as a function of the preceding terms.

Examples of Recursive Definitions

The **Fibonacci sequence** $1, 1, 2, 3, 5, 8, \dots$ can be defined as:

$$\text{Fib}(1) = 1$$

$$\text{Fib}(2) = 1$$

$$\text{Fib}(n) = \text{Fib}(n - 1) + \text{Fib}(n - 2)$$

The **factorial** function $n! = n \cdot (n - 1) \cdot \dots \cdot 3 \cdot 2 \cdot 1$ can be defined as:

$$0! = 1$$

$$n! = n \cdot (n - 1)!$$

Exercises

1. The **natural numbers** are $0, 1, 2, \dots$. Find a recursive definition for $\text{Nat}(n)$, the n th natural number.
2. The n th **triangle number** equals the sum of the first n numbers. Find a recursive definition for $\text{Tri}(n)$, the n th triangle number.

